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Slavnov-Taylor parameterization for the quantum restoration of BRST symmetries in anomaly-free gauge theories

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Abstract: It is shown that the problem of the recursive restoration of the Slavnov-Taylor (ST) identities at the quantum level for anomaly-free gauge theories is equivalent to the problem of parameterizing the local approximation to the quantum effective action in terms of ST functionals, associated with the cohomology classes of the classical linearized ST operator $\mathcal{S}_0$. The ST functionals of dimension ≤ 4 correspond to the invariant counterterms, those of dimension > 4 generate the non-symmetric counterterms upon projection on the action-like sector. At orders higher than one in the loop expansion there are additional contributions to the non-invariant counterterms, arising from known lower order terms. They can also be parameterized by using the ST functionals. We apply the method to Yang-Mills theory in the Landau gauge with an explicit mass term introduced in a BRST-invariant way via a BRST doublet. Despite being non-unitary, this model provides a good example where the method devised in the paper can be applied to derive the most general solution for the action-like part of the quantum effective action, compatible with the fulfillment of the ST identities, the ghost equation, the anti-ghost equation and the B-equation, to all orders in the loop expansion. We also provide the full dependence of the solution on the normalization conditions, that have to be supplemented in order to fix the parameters left free in the symmetric quantum effective action after the imposition of the ST identities and of the other relevant symmetries of the model.

Keywords: Renormalization and Regularization, BRST Quantization, BRST Symmetry.

Contents

1. Introduction

In the BRST quantization of gauge theories the BRST invariance of the gauge-fixed classical action is translated at the quantum level into the fulfillment of the Slavnov-Taylor (ST) identities for the quantum effective action [1, 2, 3, 4, 7]. As is well-known, the ST identities cannot be restored in the case of anomalous gauge theories [7]. That is, there does not exist any choice of finite non-symmetric counterterms, allowing to restore the ST identities at the quantum level. As a result, the corresponding quantum theories fail to comply with unitarity requirements [2]–[6].

Even for non-anomalous theories the regularization procedure, required to deal with UV divergences, may break the ST identities at the quantum level. Unlike those of anomalous models, these breakings are spurious, since they can be removed by a suitable choice of finite non-symmetric counterterms, order by order in the loop expansion. The existence of the non-symmetric counterterms, allowing to restore the ST identities at the quantum level, and the complete algebraic characterization of the action-like symmetric counterterms have been analyzed in a regularization-independent way within the framework of Algebraic Renormalization (for a review see [7]). Along these lines pure Yang-Mills theory has been discussed for general gauge-fixing condition in [10]. Yang-Mills in the background field formulation is considered in [11]. The complete renormalization structure of the Standard Model has been studied in [8, 9]. Finally, the Minimal Supersymmetric Standard Model has been analyzed in [12].

In the Standard Model and in the Minimal Supersymmetric Standard Model no regularization scheme is known to preserve *ab initio* and to all orders in the loop expansion the ST identities, due to the presence of the γ_5 matrix and of the completely antisymmetric tensor $\epsilon_{\mu\nu\rho\sigma}$.

We point out that the methods used in Algebraic Renormalization in order to guarantee that the model is anomaly-free and to characterize by algebraic means the symmetric

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counterterms and the corresponding normalization conditions [7] do not yield *per se* the non-symmetric counterterms, required in the explicit computations of the Feynman amplitudes at the loop level.

The problem of the construction of the non-symmetric counterterms was addressed in [14, 15]. Under the assumption that the ST identities have been recursively restored up to order $n-1$ in the loop expansion, by the Quantum Action Principle [22]–[26] the most general n-th order ST breaking functional $\Delta^{(n)}$ is a local integrated functional in the fields, the antifields and their derivatives with bounded dimension. $\Delta^{(n)}$ can be explicitly computed and the non-symmetric counterterms, designed to reabsorb $\Delta^{(n)}$, can be worked out. This was done for the abelian Higgs-Kibble model in [14].

In [15] it was shown that the computation of the n-th order breaking terms can actually be avoided: for anomaly-free models and under the assumptions that the ST identities have been recursively restored up to order $n-1$, the direct imposition of the n-th order ST identities at the level of the symmetric n-th order effective action is shown to be equivalent, in the absence of IR problems, to the solution of a set of linear equations, whose unknowns are the n-th order coefficients of the action-like monomials. These coefficients turn out to be functions of the n-th order superficially convergent Feynman amplitudes and known lower-order contributions. The solution is non-unique: the most general solution is obtained by adding all possible symmetric action-like counterterms to a particular solution of the set of linear equations. The ambiguities must be fixed by supplementing suitable normalization conditions [14, 15].

This method has a very general range of applicability and does not rely on the existence of a nilpotent classical linearized ST operator. Indeed in [15] it was applied to the restoration of the ST identities in the massive abelian Higgs-Kibble model, where the linearized classical ST operator is not nilpotent, due to the presence of an explicit mass term for the abelian gauge field.

In [16]–[19] this technique was applied to the case of Standard Model processes. Supersymmetric models have been analyzed in [27, 28].

In this paper we will analyze an alternative algorithm for the computation of the non-symmetric counterterms, required in order to recursively restore the ST identities, which can be applied whenever the linearized classical ST operator $\mathcal{S}_0$ is nilpotent. This method provides a geometrical characterization of the non-invariant counterterms: it is shown that the recursive imposition of the ST identities can be equivalently formulated as a parameterization problem for the local approximation of the n-th order symmetric quantum effective action $\mathrm{I}\Gamma^{(n)}$ in terms of ST functionals, associated with the cohomology classes of the classical linearized ST operator $\mathcal{S}_0$.

This allows for an effective characterization of both the symmetric and the non-symmetric counterterms, needed to restore the ST identities at order $n \geq 1$ in the loop expansion, as it will be explained in section 2.

It turns out that solving the parameterization problem is easier than directly solving the problem of the recursive restoration of the STI. In particular, the parameterization problem is more suited to be implemented via symbolic computation [35].

In this paper we focus on the geometrical aspects of this procedure. We analyze the simple case of Yang-Mills theory in the Landau gauge with an explicit mass term introduced in a BRST-invariant way via a BRST doublet [36]. Despite being non-unitary at non-zero mass, this model provides a good example where the method described in this paper can be applied to derive the most general solution for the action-like part of the quantum effective action, compatible with the fulfillment of the ST identities, the ghost equation, the antighost equation and the B-equation, to all orders in the loop expansion. In particular, since all dimensionful parameters enter into the classical action via a BRST-exact term, many simplifications arise in the solution of the parameterization problem.

The analysis of those models where dimensionful parameters are introduced in the classical action via cohomologically non-trivial terms (like in spontaneously broken gauge theories) only involves additional computational complications in the solution of the parameterization problem, which are better dealt with by computerized algorithms [35]. In particular, it turns out that the construction of the action-like part of $\mathrm{I\Gamma}^{(n)}$ in terms of the ST functionals remains comparatively simple [35].

The paper is organized as follows. In section 2 we discuss how the recursive restoration of the ST identities for anomaly-free gauge theories can be translated into a parameterization problem for the local approximation to the symmetric quantum effective action in terms of suitable ST functionals. These ST functionals are associated with the cohomology classes of $\mathcal{S}_0$ in the space of local functionals without action-like power-counting restriction. This in turn provides useful simplifications in the computation of the non-invariant counterterms, needed to restore the ST identities at higher order in the loop expansion. Moreover, this provides a geometrical insight into the structure of the local part of the symmetric quantum effective action, relevant for the restoration of the ST identities. By using the method described in section 2 the close connection between the ST breaking terms and the existence of mass scales in the theory becomes apparent.

In section 3 we discuss Yang-Mills theory in the Landau gauge with an explicit mass term introduced in a BRST-invariant way via a BRST doublet. We derive the most general form of the action-like part of the quantum effective action, compatible with the fulfillment of the ST identities, the ghost equation, the antighost equation and the B-equation, to all order in the loop expansion, and discuss how to recursively construct the non-symmetric n-th order counterterms, needed to restore the ST identities possibly broken by the regularized n-th order quantum effective action.

Finally conclusions are presented in section 4.

2. ST parameterization of the quantum effective action

In this section we will describe an effective method for the restoration at the quantum level of the Slavnov-Taylor (ST) identities

$$\mathcal{S}(\mathrm{I\Gamma}) \equiv (\mathrm{I\Gamma}, \mathrm{I\Gamma}) = \sum_i \int d^4x \, \frac{\delta \mathrm{I\Gamma}}{\delta \Phi_i^*} \frac{\delta \mathrm{I\Gamma}}{\delta \Phi_i} = 0 \tag{2.1}$$

for anomaly-free gauge theories.

The symmetric quantum effective action $\mathrm{I\!\Gamma}$ in eq. (2.1) is a functional of the fields Φ_i and of the corresponding antifields Φ_i^*, coupled in the BRST-invariant classical action $\mathrm{I\!\Gamma}^{(0)}$ to the BRST variation of the fields Φ_i. For the sake of definiteness, we will restrict ourselves to the case of closed gauge algebras. The fields Φ_i include the gauge and matter fields, collectively denoted by φ_k, the ghost fields ω^α and the fields $\bar{\omega}^\alpha, B^\alpha$, belonging to the non-minimal sector needed to fix the gauge [20]. Their BRST transformations are

$$s\varphi_k = R_k^\alpha[\varphi]\omega_\alpha\,, \qquad s\omega_\alpha = \frac{1}{2}F_\alpha^{\beta\gamma}\omega_\beta\omega_\gamma\,, \qquad s\bar{\omega}^\alpha = B^\alpha\,, \qquad sB^\alpha = 0\,. \tag{2.2}$$

The constants $F_\alpha^{\beta\gamma}$ are antisymmetric in the indices β, γ and are related to R_k^α by the algebra

$$R_j^\alpha[\varphi]\frac{\delta R_k^\beta[\varphi]}{\delta\varphi_j} - R_j^\beta[\varphi]\frac{\delta R_k^\alpha[\varphi]}{\delta\varphi_j} = F_\gamma^{\alpha\beta}R_k^\gamma[\varphi]\,. \tag{2.3}$$

The Jacobi identity

$$F_\alpha^{\beta[\gamma}F_\beta^{\delta\sigma]} = 0 \tag{2.4}$$

holds true for the structure constants $F_\gamma^{\alpha\beta}$. We assign mass dimension $+1$ to the fields ω^α and $\bar{\omega}^\alpha$ and mass dimension $+2$ to B^α.

From the point of view of renormalization, the antifields Φ_i^* are only needed for those fields whose BRST variation is non-linear in the quantum fields [13]. However, in order to keep the notation uniform, we assume that an antifield has been introduced for each field of the model. This allows to write the ST identities in the form of eq. (2.1).

The symmetric quantum effective action $\mathrm{I\!\Gamma}$ admits an expansion in the loop parameter $\hbar$:

$$\mathrm{I\!\Gamma} = \sum_{n=0}^{\infty}\mathrm{I\!\Gamma}^{(n)}\,. \tag{2.5}$$

In the above equation $\mathrm{I\!\Gamma}^{(n)}$ stands for the coefficient of $\mathrm{I\!\Gamma}$ of order n in the $\hbar$ expansion.

Let us assume that the ST identities in eq. (2.1) have been fulfilled up to order $n-1$, so that

$$\mathcal{S}(\mathrm{I\!\Gamma})^{(j)} = 0\,, \qquad j = 0, 1, \ldots, n-1\,. \tag{2.6}$$

At the n-th order in the loop expansion the ST identities read

$$\mathcal{S}_0(\mathrm{I\!\Gamma}^{(n)}) = -\sum_{k=1}^{n-1}\left(\mathrm{I\!\Gamma}^{(n-k)}, \mathrm{I\!\Gamma}^{(k)}\right), \tag{2.7}$$

where $\mathcal{S}_0$ is the classical linearized ST operator

$$\mathcal{S}_0 = \sum_i \int d^4x \left(\frac{\delta\mathrm{I\!\Gamma}^{(0)}}{\delta\Phi_i^*}\frac{\delta}{\delta\Phi_i} + \frac{\delta\mathrm{I\!\Gamma}^{(0)}}{\delta\Phi_i}\frac{\delta}{\delta\Phi_i^*}\right). \tag{2.8}$$

The bracket in the r.h.s. of eq. (2.7) is given by

$$(X, Y) = \sum_i \int d^4x\, \frac{\delta X}{\delta\Phi_i^*}\frac{\delta Y}{\delta\Phi_i}\,. \tag{2.9}$$

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Whenever the linearized classical ST operator $\mathcal{S}_0$ is nilpotent, we will prove that the problem of the fulfillment of anomaly-free ST identities at the n-th order in the loop expansion can be equivalently formulated as the problem of parameterizing the local approximation to the quantum effective action in terms of ST functionals, associated with the cohomology classes of the classical linearized ST operator. Upon projection of $\mathrm{I}\Gamma^{(n)}$ on the action-like sector, we construct the full n-th order action-like part of the symmetric quantum effective action, fulfilling the ST identities up to order n.

The $\mathcal{S}_0$-invariants with dimension ≤ 4 provide the usual symmetric counterterms. Their coefficients are not fixed by the ST identities and enter as free parameters into $\mathrm{I}\Gamma^{(n)}$. There may exist additional relations between these parameters, stemming from additional symmetries obeyed by the quantum effective action like for instance the ghost equation, discrete symmetries like C-parity, etc. Once these relations have been taken into account, the remaining free parameters must be fixed by supplying a suitable set of normalization conditions [7].

We will first briefly review the construction given in [15], since this helps clarifying the novel features of the method we propose in this paper and its range of applicability. We are only concerned here with the construction of the action-like part of $\mathrm{I}\Gamma$; the first non-vanishing order in the loop expansion of the ST identities is local, due to the QAP. Hence we can restrict ourselves to the space of local functionals and consider the effective local approximation to $\mathrm{I}\Gamma$. That is, we associate to $\mathrm{I}\Gamma^{(n)}$ a formal power series given by an infinite sum of local Lorentz-invariant functionals. We will denote the series by $\mathrm{I}\Gamma^{(n)}$ itself:

$$\mathrm{I}\Gamma^{(n)} = \sum_j \int d^4x\, m_j^{(n)} \mathcal{M}_j(x)\,. \tag{2.10}$$

In the absence of IR problems, as it happens for the massive abelian Higgs-Kibble model analyzed in [15], $\mathrm{I}\Gamma^{(n)}$ in eq. (2.10) is obtained by performing the Taylor expansion of the symmetric 1-PI amplitudes in the independent external momenta around zero.

The Lorentz-scalar monomials $\mathcal{M}_j(x)$ in the fields and external sources (and their derivatives) have to comply with all unbroken symmetries of the theory and must have ghost number zero. $\mathcal{M}_j(x)$ are chosen to be linearly independent. They span the vector space $\mathcal{V}$, to which $\mathrm{I}\Gamma^{(n)}$ in eq. (2.10) belongs. Notice that we do not impose any power-counting restriction in the r.h.s. of eq. (2.10): the basis $\{\mathcal{M}_j\}_{j\in\mathbf{N}}$ contains elements with arbitrary positive dimension. Once the basis $\{\mathcal{M}_j\}_{j\in\mathbf{N}}$ has been chosen, the coefficients $m_j^{(n)}$ in eq. (2.10) are uniquely determined. We denote by $\gamma_j^{(n)}$ those coefficients in eq. (2.10), associated to monomials $\mathcal{M}_j(x)$ with dimension strictly greater than four. They correspond to superficially convergent Feynman amplitudes. We denote by $\xi_j^{(n)}$ the coefficients of the monomials $\mathcal{M}_j$ with dimension less or equal than four (action-like part of $\mathrm{I}\Gamma^{(n)}$).

We always assume that the ST identities have been restored up to order $n-1$, i.e.

$$\mathcal{S}(\mathrm{I}\Gamma)^{(j)} = 0\,, \qquad j = 0, 1, \ldots, n-1\,. \tag{2.11}$$

The n-th order ST identities read:

$$\mathcal{S}(\mathrm{I}\Gamma)^{(n)} = \mathcal{S}_0(\mathrm{I}\Gamma^{(n)}) + \sum_{j=1}^{n-1} \left(\mathrm{I}\Gamma^{(n-j)}, \mathrm{I}\Gamma^{(j)}\right) = 0\,. \tag{2.12}$$

The brackets in eq. (2.12) only involve $\mathrm{I\!\Gamma}^{(i)}$ with $i < n$, which are known and fulfill eq. (2.11). The unknown quantities are the action-like terms of $\mathrm{I\!\Gamma}^{(n)}$ (i.e. the monomials with dimension less or equal to 4 with the correct symmetry properties), which we denote by $\Xi^{(n)}$. The regularized action $\Gamma^{(n)}$ is constructed by using the counterterms $\Xi^{(j)}$, $j < n$ and it is finite. We can expand $\Xi^{(n)}$ on a basis of integrated Lorentz-invariant monomials with dimension ≤ 4, as follows:

$$\Xi^{(n)} = \sum_j \int d^4x \, \xi_j^{(n)} \mathcal{M}^j(x) \,. \tag{2.13}$$

$\mathcal{S}(\mathrm{I\!\Gamma})^{(n)}$ is an element of the vector space $\mathcal{W}$ spanned by all possible linearly independent Lorentz-invariant monomials in the fields and the external sources and their derivatives with ghost number $+1$, with possibly additional symmetry properties dictated by the relevant symmetries of the model. We denote by $\{\mathcal{N}_i(x)\}_{i=1,2,3,\ldots}$ a basis for $\mathcal{W}$. Then we can insert the decomposition in eq. (2.10) into eq. (2.12) and get

$$\begin{aligned}\mathcal{S}(\mathrm{I\!\Gamma})^{(n)} &= \sum_j m_j^{(n)} \mathcal{S}_0 \left(\int d^4x \, \mathcal{M}_j \right) + \\ &\quad + \sum_{i=1}^{n-1} \sum_{jj'} m_j^{(i)} m_{j'}^{(n-i)} \left(\int d^4x \mathcal{M}_j(x), \int d^4x' \mathcal{M}_{j'}(x') \right) .\end{aligned} \tag{2.14}$$

There exist coefficients $a_r^j, b_{kr}^{jj'}$ (uniquely fixed by the choice of $\mathcal{N}_i(x)$ and by the action of $\mathcal{S}_0$) such that

$$\mathcal{S}_0 \left(\int d^4x \, \mathcal{M}_j \right) = \int d^4x \sum_r a_r^j \mathcal{N}_r(x) \,, \tag{2.15}$$

$$\left(\int d^4x \mathcal{M}_j(x), \int d^4x' \mathcal{M}_{j'}(x') \right) = \int d^4x \sum_r b_r^{jj'} \mathcal{N}_r(x) \,. \tag{2.16}$$

Then eq. (2.14) becomes

$$\sum_j a_r^j m_j^{(n)} + \sum_{i=1}^{n-1} \sum_{jj'} m_j^{(i)} m_{j'}^{(n-i)} b_r^{jj'} = 0 \,, \qquad r = 0, 1, 2 \ldots \tag{2.17}$$

For r such that $\dim \mathcal{N}_r(x) > 5$ (with the conventions on the dimensions given in the Introduction), eq. (2.17) is an identity by the virtue of the Quantum Action Principle (QAP). For r such that $\dim \mathcal{N}_r \leq 5$ eq. (2.17) defines an inhomogeneous linear problem in the unknowns $\xi_j^{(n)}$. In the absence of anomalies, one can solve eq. (2.17) by expressing the coefficients $\xi_j^{(n)}$ in terms of the coefficients $\gamma_j^{(n)}$ of the n-th order superficially convergent Feynman amplitudes and of the lower-order coefficients $m_j^{(l)}$, $l < n$. That is, one can construct $\Xi^{(n)}$ from the superficially convergent part of $\mathrm{I\!\Gamma}^{(n)}$ and from lower order contributions $\mathrm{I\!\Gamma}^{(l)}$, $l < n$.

The solution is not unique, due to the fact that the kernel of $\mathcal{S}_0$ is not empty. The ambiguities are parameterized by the action-like symmetric counterterms, possibly restricted by further symmetries of the quantum effective action like C-parity, the ghost equation, and so on. These ambiguities must be fixed by providing a suitable set of normalization conditions.

This procedure has the advantage that it can be applied to very general situations. Indeed one can also drop the requirement of nilpotency of $\mathcal{S}_0$ and still the linear problem in eq. (2.17) remains well-defined. We remark that in [15] this method was applied to the massive abelian Higgs-Kibble model, whose associated classical linearized ST operator $\mathcal{S}_0$ is not nilpotent, due to the explicit mass term for the abelian gauge field.

In those cases where $\mathcal{S}_0$ is a truly nilpotent differential operator, it turns out that it is possible to write directly the solution to eq. (2.17) by exploiting the corresponding homogeneous equation, to be solved in a space of local functionals not restricted by power-counting. Since the latter defines a cohomological problem in the space of local functionals, the powerful techniques of homological perturbation theory [29, 30, 31, 32, 34] can be used to obtain the required solution. The action-like part of $\mathrm{I\!\Gamma}^{(n)}$ is then recovered by projecting the solution onto the action-like sector.

In what follows $\Gamma^{(n)}$ stands for the n-th order regularized vertex functional. We first discuss the one-loop order. Eq. (2.12) reads at the one-loop level:

$$\mathcal{S}_0(\mathrm{I\!\Gamma}^{(1)}) = 0\,. \tag{2.18}$$

$\mathrm{I\!\Gamma}^{(1)}$ in eq. (2.18) denotes the general solution fulfilling the one-loop ST identities. We do not impose for the moment any normalization condition. Hence $\mathrm{I\!\Gamma}^{(1)}$ will depend on a set of free parameters, in one-to-one correspondence with the independent symmetric counterterms of the model at hand.

In the absence of anomalies, we know that for any regularized effective action $\Gamma^{(1)}$ there exists a local action-like counterterm functional $\Upsilon^{(1)}$ such that

$$\mathrm{I\!\Gamma}^{(1)} = \Gamma^{(1)} + \Upsilon^{(1)}\,. \tag{2.19}$$

Since $\Upsilon^{(1)}$ is action-like, $(1-t^4)\Upsilon^{(1)} = 0$. $t^4 X$ denotes the projector on the action-like sector of the functional X. From eq. (2.19) we get that

$$(1-t^4)\Gamma^{(1)} = (1-t^4)\mathrm{I\!\Gamma}^{(1)}\,. \tag{2.20}$$

That is, the non action-like part of $\mathrm{I\!\Gamma}^{(1)}$ is fixed by the regularized one-loop effective action. The direct construction of the action-like part of $\mathrm{I\!\Gamma}^{(1)}$ proceeds as follows. Since we are only concerned with the construction of $t^4\mathrm{I\!\Gamma}^{(1)}$ and the first non-vanishing order of the ST identities in the loop expansion is local by the QAP, we can restrict ourselves to the local approximation given by eq. (2.10). Then we want to solve

$$\mathcal{S}_0(\mathrm{I\!\Gamma}^{(1)}) = \mathcal{S}_0\left(t^4\mathrm{I\!\Gamma}^{(1)} + (1-t^4)\mathrm{I\!\Gamma}^{(1)}\right) = 0 \tag{2.21}$$

in the unknown $t^4\mathrm{I\!\Gamma}^{(1)}$ (action-like part of $\mathrm{I\!\Gamma}^{(1)}$). By eq. (2.21) we see that the difference between $t^4\mathrm{I\!\Gamma}^{(1)}$ and $-(1-t^4)\mathrm{I\!\Gamma}^{(1)}$ must belong to the kernel of $\mathcal{S}_0$. Since we are dealing with

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local functionals, a full characterization of the kernel of $\mathcal{S}_0$ is available by means of cohomological techniques [29, 30, 31, 32, 34]. We stress that these techniques are independent of the power-counting restrictions. This plays an essential rôle here.

Any element $\mathcal{K}$ in the kernel of $\mathcal{S}_0$ in the sector with ghost number zero can be written as

$$\mathcal{K} = \sum_j \lambda_j^{(1)} \Lambda_j + \mathcal{S}_0 \left(\mathcal{K}_{-\mathbf{1}}^{(1)} \right) . \tag{2.22}$$

where the sum over j is a sum over the cohomology classes Λ_j of $\mathcal{S}_0$ (not restricted by power-counting), $\lambda_j^{(1)}$ are c-number coefficients and $\mathcal{K}_{-\mathbf{1}}^{(1)}$ is a local functional with ghost number -1. We remark that Λ_j are explicitly known for many gauge theories [29, 30, 31, 32]. Recently this problem has also been solved for a wide class of four-dimensional supersymmetric models [33]. From eq. (2.21) and eq. (2.22) we conclude that

$$t^4 \mathrm{I\!\Gamma}^{(1)} = -(1-t^4)\mathrm{I\!\Gamma}^{(1)} + \sum_j \lambda_j^{(1)} \Lambda_j + \mathcal{S}_0 \left(\mathcal{K}_{-\mathbf{1}}^{(1)} \right) . \tag{2.23}$$

Now we apply the projector t^4 to both sides of eq. (2.23) (this takes into account the dimension bounds on the functionals) and we get

$$\begin{aligned} \Xi^{(1)} &= t^4 \mathrm{I\!\Gamma}^{(1)} = \sum_j \lambda_j^{(1)} t^4 \Lambda_j + t^4 \left(\mathcal{S}_0 (\mathcal{K}_{-\mathbf{1}}) \right) \\ &= \sum_j \lambda_j^{(1)} t^4 \Lambda_j + t^4 \left(\mathcal{S}_0 \left(\sum_r \rho_r^{(1)} \mathcal{K}_{-\mathbf{1},r} \right) \right) . \end{aligned} \tag{2.24}$$

In the second line of the above equation we have expanded the functional $\mathcal{K}_{-\mathbf{1}}^{(1)}$ on a basis $\{\mathcal{K}_{-\mathbf{1},r}\}$ for the sector with ghost number -1.

The formula in eq. (2.24) provides an explicit solution for $t^4\mathrm{I\!\Gamma}^{(1)}$ in terms of local functionals belonging to the kernel of $\mathcal{S}_0$. The coefficients $\lambda_j^{(1)}$, associated with representatives Λ_j whose dimension is ≤ 4,[1] are not constrained by the ST identities. They are free parameters entering into the solution. They may not be all independent, due to the existence of additional symmetries of the model (like the ghost equation and discrete symmetries [7]). On the contrary, the coefficients $\lambda_j^{(1)}$ of those invariants whose dimension is greater than 4 are not free: they can be expressed in terms of the one-loop superficially convergent Feynman amplitudes entering into $(1-t^4)\Gamma^{(1)} = (1-t^4)\mathrm{I\!\Gamma}^{(1)}$. In the same way, the coefficients $\rho_r^{(1)}$, associated to those $\mathcal{S}_0$-invariants $\mathcal{S}_0(\mathcal{K}_{-\mathbf{1},r})$ whose dimension is ≤ 4, are not constrained by the ST identities and are free parameters entering into the solution. Again there may exist relations between them, due to additional symmetries of the model. The coefficients $\rho_r^{(1)}$, associated to those $\mathcal{S}_0$-invariants $\mathcal{S}_0(\mathcal{K}_{-\mathbf{1},r})$ with dimension > 4, are fixed by the known non-action-like part of $\mathrm{I\!\Gamma}^{(1)}$ $(1-t^4)\mathrm{I\!\Gamma}^{(1)} = (1-t^4)\Gamma^{(1)}$. We notice that the coefficients $\xi_j^{(1)}$ in eq. (2.13) are in general functions of the coefficients $\lambda_j^{(1)}$ and $\rho_r^{(1)}$ in

[1] If $P = \sum_k \mathcal{Q}_k$ is a polynomial in the fields, the external sources and their derivatives, decomposed into the sum over the monomials $\mathcal{Q}_k$, we define the dimension of $\mathcal{P}$ as the maximum of the dimensions of $\mathcal{Q}_k$.

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eq. (2.24). The non-symmetric counterterms, entering into $t^4\mathrm{I\!\Gamma}^{(1)}$, are given by

$$\Xi^{(1)}_{\rm n.s.} = {\sum_j}' \lambda^{(1)}_j t^4 \Lambda_j + t^4 \left(\mathcal{S}_0 \left({\sum_r}' \rho^{(1)}_r \mathcal{K}_{-\mathbf{1},r} \right) \right) , \tag{2.25}$$

where the primed sum over j is restricted to those invariants Λ_j with dimension > 4 and the primed sum over r is restricted to those invariants $\mathcal{S}_0(\mathcal{K}_{-\mathbf{1},r})$ with dimension > 4.

If the classical action contains dimensionful parameters like masses it can happen [35] that $\Xi^{(1)}_{\rm n.s.}$ is non-zero. As an example, this accounts for the dependence of the non-invariant counterterms on the Higgs v.e.v. v in the case of the abelian Higgs-Kibble model analyzed in [14].

The method of the parameterization in terms of ST functionals allows to derive easily the structure of the symmetric solution $\mathrm{I\!\Gamma}^{(1)}$. Moreover, it provides a direct control of the dependence on the normalization conditions chosen.

Let us now analyze how this procedure can be extended to higher orders. The n-th order ST identities are

$$\mathcal{S}_0(\mathrm{I\!\Gamma}^{(n)}) + \sum_{k=1}^{n-1} \left(\mathrm{I\!\Gamma}^{(k)}, \mathrm{I\!\Gamma}^{(n-k)} \right) = 0 \, . \tag{2.26}$$

Due to the fact that $\mathcal{S}_0^2 = 0$, by applying $\mathcal{S}_0$ to both sides of eq. (2.26) we get

$$\mathcal{S}_0 \left(\sum_{k=1}^{n-1} \left(\mathrm{I\!\Gamma}^{(k)}, \mathrm{I\!\Gamma}^{(n-k)} \right) \right) = 0 \, . \tag{2.27}$$

By virtue of eq. (2.27) we get that

$$\sum_{k=1}^{n-1} \left(\mathrm{I\!\Gamma}^{(k)}, \mathrm{I\!\Gamma}^{(n-k)} \right) = \sum_{j \in \mathcal{J}} \lambda^{(n)}_j \mathcal{A}_j + \mathcal{S}_0 \left(\tilde{\zeta}^{(n)}_{\mathbf{0}} \right) . \tag{2.28}$$

In the above equation the sum over $j \in \mathcal{J}$ is a sum over the representatives of the non-trivial cohomology classes of $\mathcal{S}_0$ with ghost number +1 (anomalies) and $\tilde{\zeta}^{(n)}_{\mathbf{0}}$ is a functional with ghost number zero.

Let us assume that no anomalies are present due to purely algebraic reasons, so that the set $\mathcal{J}$ is empty. Hence eq. (2.28) reduces to

$$\sum_{k=1}^{n-1} \left(\mathrm{I\!\Gamma}^{(k)}, \mathrm{I\!\Gamma}^{(n-k)} \right) = \mathcal{S}_0 \left(\tilde{\zeta}^{(n)}_{\mathbf{0}} \right) \tag{2.29}$$

and eq. (2.26) now becomes

$$\mathcal{S}_0 \left(\mathrm{I\!\Gamma}^{(n)} + \tilde{\zeta}^{(n)}_{\mathbf{0}} \right) = 0 \, , \tag{2.30}$$

which parallels eq. (2.18). Notice that $\tilde{\zeta}^{(n)}_{\mathbf{0}}$ is a known functional constructed from $\mathrm{I\!\Gamma}^{(i)}$, $i < n$, which are known. By following the same approach described before for the one-loop level we get from eq. (2.30)

$$\mathcal{S}_0 \left[t^4 \left(\mathrm{I\!\Gamma}^{(n)} + \tilde{\zeta}^{(n)}_{\mathbf{0}} \right) + (1 - t^4) \left(\mathrm{I\!\Gamma}^{(n)} + \tilde{\zeta}^{(n)}_{\mathbf{0}} \right) \right] = 0 \tag{2.31}$$

or

$$t^4\left(\mathrm{I\!\Gamma}^{(n)}+\tilde{\zeta}^{(n)}_{\mathbf{0}}\right)=-(1-t^4)\left(\mathrm{I\!\Gamma}^{(n)}+\tilde{\zeta}^{(n)}_{\mathbf{0}}\right)+\sum_j \lambda^{(n)}_j\Lambda_j+\mathcal{S}_0\left(\mathcal{K}^{(n)}_{-\mathbf{1}}\right). \tag{2.32}$$

By applying the projection operator t^4 to eq. (2.32) we get

$$t^4\mathrm{I\!\Gamma}^{(n)}=-t^4\tilde{\zeta}^{(n)}_{\mathbf{0}}+\sum_j \lambda^{(n)}_j t^4\Lambda_j+t^4\mathcal{S}_0\left(\sum_r \rho^{(n)}_r\mathcal{K}_{-\mathbf{1},r}\right), \tag{2.33}$$

which gives explicitly the action-like part of $\mathrm{I\!\Gamma}^{(n)}$ to orders $n>1$. The n-th order non-symmetric counterterms entering into $t^4\mathrm{I\!\Gamma}^{(1)}$ are given by

$$\Xi^{(n)}_{\text{n.s.}}=-t^4\tilde{\zeta}^{(n)}_{\mathbf{0}}+\sum_j{}' \lambda^{(n)}_j t^4\Lambda_j+t^4\mathcal{S}_0\left(\sum_r{}' \rho^{(n)}_r\mathcal{K}_{-\mathbf{1},r}\right) \tag{2.34}$$

where the primed sum over j is restricted to those invariants Λ_j with dimension >4 and the primed sum over r is restricted to those invariants $\mathcal{S}_0(\mathcal{K}_{-\mathbf{1},r})$ with dimension >4. Notice the appearance of non-symmetric counterterms depending on lower orders via the term $-t^4\tilde{\zeta}^{(n)}_{\mathbf{0}}$ in the r.h.s. of eq. (2.34).

Let us comment on the results of this subsection. The problem of the restoration of the ST identities has been turned into a parameterization problem for the local approximation to the symmetric quantum effective action. It can be parameterized by means of the ST functionals Λ_j and $\mathcal{S}_0(\mathcal{K}_{-\mathbf{1},r})$. The lower order contributions to the ST identities given by eq. (2.29) are parameterized via the functional $\tilde{\zeta}^{(n)}_{\mathbf{0}}$.

Then the full action-like part of the n-th order symmetric quantum effective action $t^4\mathrm{I\!\Gamma}^{(n)}$ is provided by eq. (2.33).

3. Yang-Mills theory with explicit BRST-invariant mass term

In this section we apply the method described in section 2 to Yang-Mills theory in the Landau gauge with an explicit mass term introduced in a BRST-invariant way via a BRST doublet [36]. At mass different than zero this model is not unitary. Nevertheless, it provides a good example to illustrate the method analyzed in section 2. This is due to the fact that, since all dimensionful parameters enter into the classical action via a BRST-exact term, some simplifications arise in the structure of the non-symmetric counterterms in $t^4\mathrm{I\!\Gamma}^{(n)}$ in eq. (2.25) and in eq. (2.34). In particular, there are no contributions from the Λ-type invariants to the non-symmetric counter-terms in eqs. (2.25) and (2.34). However there are explicit contributions to the non-symmetric counterterms in $t^4\mathrm{I\!\Gamma}^{(n)}$ at orders higher than one, coming from $-t^4\tilde{\zeta}^{(n)}_{\mathbf{0}}$ in eq. (2.34).

We will show how the parameterization problem for the construction of $\tilde{\zeta}^{(n)}_{\mathbf{0}}$ can be solved by using techniques of homological perturbation theory and how the method described in section 2 can be used to control the full dependence of $t^4\mathrm{I\!\Gamma}^{(n)}$ on the normalization conditions chosen.

We do not discuss here the zero-mass limit.

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We start from the classical action of Yang-Mills theory based on the simple compact gauge group G:

$$S_{\rm YM} = \int d^4x \Big\{ -\frac{1}{4g^2} G^a_{\mu\nu} G^{\mu\nu\, a} - \bar\omega^a \partial_\mu (D^\mu \omega)^a + B^a \partial A^a + \\ + A^{a*}_\mu (D^\mu\omega)^a - \omega^{*a} \frac{1}{2} f^{abc} \omega^b \omega^c + \bar\omega^{a*} B^a \Big\}. \tag{3.1}$$

The non-abelian gauge field strength $G^a_{\mu\nu}$ is

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc} A^b_\mu A^c_\nu \,, \tag{3.2}$$

while the covariant derivative D^μ_{ac} in the adjoint representation is given by

$$D^\mu_{ac} = \delta_{ac} \partial^\mu + f_{abc} A^\mu_b \,. \tag{3.3}$$

f^{abc} are the structure constants of the gauge group, g is the coupling constant of the model. ω^a are the ghost fields, $\bar\omega^a$ the antighost fields, B^a the Nakanishi-Lautrup multiplier fields.

The BRST differential s acts as follows on the fields of the model:

$$\begin{aligned} sA^a_\mu &= (D_\mu \omega)^a \,, & s\omega^a &= -\frac{1}{2} f^{abc} \omega^b \omega^c \,, \\ s\bar\omega^a &= B^a \,, & sB^a &= 0 \,. \end{aligned} \tag{3.4}$$

In $S_{\rm YM}$ we have coupled the BRST variations of $A^a_\mu, \omega^a, \bar\omega^a$ to the antifields $A^{a*}_\mu, \omega^{a*}, \bar\omega^{a*}$. The dimension of $A^a_\mu, \omega^a, \bar\omega^a$ is one, the dimension of $B^a, A^{a*}_\mu, \omega^{a*}, \bar\omega^{a*}$ is two.

The ghost number is assigned as follows:

$$\begin{aligned} gh(A^a_\mu) &= gh(B^a) = gh(\bar\omega^{a*}) = 0 \,, & gh(\omega^a) &= +1 \,, \\ gh(\bar\omega^a) &= gh(A^{a*}_\mu) = -1 \,, & gh(\omega^{a*}) &= -2 \,. \end{aligned} \tag{3.5}$$

We now introduce an anticommuting parameter $\bar\rho$ with dimension zero and ghost number -1 and its corresponding BRST partner m with dimension $+1$ and ghost number zero:

$$s\bar\rho = m \,, \qquad sm = 0 \,. \tag{3.6}$$

We consider the following classical action

$$\begin{aligned} \Gamma^{(0)} &= S_{\rm YM} + \int d^4x \, s \left(\frac{1}{2} \bar\rho m (A^a_\mu)^2 + \bar\rho m \bar\omega^a \omega^a \right) \\ &= S_{\rm YM} + \int d^4x \Big(\frac{1}{2} m^2 (A^a_\mu)^2 + m^2 \bar\omega^a \omega^a - \\ &\qquad - \bar\rho m A^a_\mu \partial^\mu \omega^a - \bar\rho m B^a \omega^a - \frac{1}{2} \bar\rho m \bar\omega^a f^{abc} \omega^b \omega^c \Big) \,. \end{aligned} \tag{3.7}$$

Notice the appearance of a mass term for the gauge bosons and the ghost fields via the s-exact term in the first line of eq. (3.7). $\Gamma^{(0)}$ is BRST-invariant:

$$s\Gamma^{(0)} = 0 \,. \tag{3.8}$$

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As a consequence, $\Gamma^{(0)}$ fulfills the ST identities:

$$\mathcal{S}(\Gamma^{(0)}) = \int d^4x \left(\frac{\delta\Gamma^{(0)}}{\delta A^{a*}_\mu} \frac{\delta\Gamma^{(0)}}{\delta A^a_\mu} + \frac{\delta\Gamma^{(0)}}{\delta\omega^{a*}} \frac{\delta\Gamma^{(0)}}{\delta\omega^a} + \frac{\delta\Gamma^{(0)}}{\delta\bar\omega^{a*}} \frac{\delta\Gamma^{(0)}}{\delta\bar\omega^a} \right) + m \frac{\partial\Gamma^{(0)}}{\partial\bar\rho} = 0 \,. \tag{3.9}$$

The linearized classical ST operator $\mathcal{S}_0$ is

$$\begin{aligned} \mathcal{S}_0 = \int d^4x \Big(& \frac{\delta\Gamma^{(0)}}{\delta A^{a*}_\mu} \frac{\delta}{\delta A^a_\mu} + \frac{\delta\Gamma^{(0)}}{\delta A^a_\mu} \frac{\delta}{\delta A^{a*}_\mu} + \frac{\delta\Gamma^{(0)}}{\delta\omega^{a*}} \frac{\delta}{\delta\omega^a} + \frac{\delta\Gamma^{(0)}}{\delta\omega^a} \frac{\delta}{\delta\omega^{a*}} \\ & + \frac{\delta\Gamma^{(0)}}{\delta\bar\omega^{a*}} \frac{\delta}{\delta\bar\omega^a} + \frac{\delta\Gamma^{(0)}}{\delta\bar\omega^a} \frac{\delta}{\delta\bar\omega^{a*}} \Big) + m \frac{\partial}{\partial\bar\rho} \,. \end{aligned} \tag{3.10}$$

$\mathcal{S}_0$ is nilpotent: $\mathcal{S}_0^2 = 0$. We remark that the full dependence of $\Gamma^{(0)}$ on $\bar\rho$ can be reabsorbed by performing the following antifield redefinitions:

$$\begin{aligned} A^{a*'}_\mu &= A^{a*}_\mu - \bar\rho m A^a_\mu \,, \\ \omega^{a*'} &= \omega^{a*} + \bar\rho m \bar\omega^a \,, \\ \bar\omega^{a*'} &= \bar\omega^{a*} - \bar\rho m \omega^a \,. \end{aligned} \tag{3.11}$$

In the new variables $\Gamma^{(0)}$ becomes

$$\begin{aligned} \Gamma^{(0)} = \int d^4x \Big\{ & -\frac{1}{4g^2} G^a_{\mu\nu} G^{\mu\nu\, a} - \bar\omega^a \partial_\mu (D^\mu\omega)^a + B^a \partial A^a + \\ & + A^{a*'}_\mu (D^\mu\omega)^a - \omega^{*a'} \frac{1}{2} f^{abc} \omega^b \omega^c + \bar\omega^{a*'} B^a + \\ & + \frac{1}{2} m^2 (A^a_\mu)^2 + m^2 \bar\omega^a \omega^a \Big\} \,. \end{aligned} \tag{3.12}$$

$\Gamma^{(0)}$ in eq. (3.12) obeys a set of additional identities:

$$\frac{\partial\Gamma^{(0)}}{\partial\bar\rho} = 0 \,, \tag{3.13}$$

the B-equation

$$\frac{\delta\Gamma^{(0)}}{\delta B^a} = \partial A^a + \bar\omega^{a*'} \,, \tag{3.14}$$

the ghost equation

$$\frac{\delta\Gamma^{(0)}}{\delta\bar\omega^a} + \partial^\mu \frac{\delta\Gamma^{(0)}}{\delta A^{a*'}_\mu} = m^2 \omega^a \,, \tag{3.15}$$

and the anti-ghost equation

$$\int d^4x \left(\frac{\delta\Gamma^{(0)}}{\delta\omega^a} - f^{abc} \bar\omega^b \frac{\delta\Gamma^{(0)}}{\delta B^c} \right) = \int d^4x \left(m^2 \bar\omega^a - f^{abc} A^{b*'}_\mu A^{\mu c} + f^{abc} \omega^{*b'} \omega^c \right) . \tag{3.16}$$

The B-equation and the ghost equation are local, while the anti-ghost equation is an integrated equation. We notice that the breaking terms in the r.h.s. of eqs. (3.15) and (3.16) are linear in the quantum fields.

All these functional identities can be restored at the quantum level by applying the standard methods dicussed e.g. in [7]. Therefore we assume from now on that both the regularized quantum effective action Γ and the symmetric quantum effective action $\rm I\!\Gamma$ fulfill the following relations:

$$\frac{\partial \Sigma^{(j)}}{\partial \bar\rho} = 0\,, \qquad j \geq 1\,, \tag{3.17}$$

$$\frac{\delta \Sigma^{(j)}}{\delta B^a} = 0\,, \qquad j \geq 1\,, \tag{3.18}$$

$$\frac{\delta \Sigma^{(j)}}{\delta \bar\omega^a} + \partial^\mu \frac{\delta \Sigma^{(j)}}{\delta A^{a*'}_\mu} = 0\,, \qquad j \geq 1\,, \tag{3.19}$$

$$\int d^4x \left(\frac{\delta \Sigma^{(j)}}{\delta \omega^a} - f^{abc} \bar\omega^b \frac{\delta \Sigma^{(j)}}{\delta B^c} \right) = \int d^4x\, \frac{\delta \Sigma^{(j)}}{\delta \omega^a} = 0\,, \qquad j \geq 1\,. \tag{3.20}$$

In the above equations Σ stands both for Γ and $\rm I\!\Gamma$. In eq. (3.20) we have used eq. (3.18). A formal proof of the validity of the identities (3.17)-(3.20) for $\rm I\!\Gamma$ can be achieved by showing that there are no anomalies if the vertex functional is restricted by eqs. (3.17)-(3.20). This can be done by applying the standard methods described for instance in [7]. Let us remark that, since $\bar\rho$ and m enter as a BRST doublet, they cannot modify the cohomological properties of the model at $\bar\rho = m = 0$.

From eq. (3.17) we see that $\Gamma^{(j)}$ and $\rm I\!\Gamma^{(j)}$, $j \geq 1$ are independent of $\bar\rho$. From eq. (3.18) we conclude that $\Gamma^{(j)}$ and $\rm I\!\Gamma^{(j)}$, $j \geq 1$, are independent of B^a. From eq. (3.19) we see that $\Gamma^{(j)}$ and $\rm I\!\Gamma^{(j)}$, $j \geq 1$ depends on $\bar\omega^a$ through the combination

$$\hat{A}^{*a'}_\mu = A^{a*'}_\mu + \partial_\mu \bar\omega^a\,. \tag{3.21}$$

3.1 One-loop level

By following the method outlined in section 2 we construct the most general solution to the action-like part of the symmetric quantum effective action $\rm I\!\Gamma^{(1)}$ at one-loop level, fulfilling the first order ST identities:

$$\mathcal{S}_0(\rm I\!\Gamma^{(1)}) = 0\,. \tag{3.22}$$

Since $\rm I\!\Gamma^{(1)}$ is independent of $\bar\rho$ by virtue of eq. (3.17), we only need to consider the restriction of $\mathcal{S}_0$ to the space of $\bar\rho$-independent functionals. By explicit computation one can verify that

$$\mathcal{S}_0(\Phi^{*'}) = \frac{\delta \Gamma^{(0)}_{m=0}}{\delta \Phi}\,, \tag{3.23}$$

for $\Phi^{*'} = \hat{A}^{a*'}_\mu, \omega^{a*'}, \bar\omega^{a*'}$. In the primed variables the operator $\mathcal{S}_0$ has definite degree $+1$ with respect to the grading induced by the dimension of the fields and the antifields. The assignment of dimensions to the primed variables is as follows: $\hat{A}^{a*'}_\mu$, $\omega^{a*'}$, $\bar\omega^{a*'}$ have dimension 2, i.e. the dimension of the primed antifield is the same as the one of the corresponding unprimed antifield.

Eq. (2.24) now gives the complete structure of the action-like part of $\rm I\!\Gamma^{(1)}$ in terms of the invariants Λ_j and $\mathcal{S}_0(\mathcal{K}_{-\mathbf{1},r})$. The classification of the invariants Λ_j, relevant for the present model, can be obtained according to the results of [7, 29, 30, 31, 32]. In particular, since there are no abelian factors, no cohomologically non-trivial invariants, involving the external sources $\hat{A}_a^{\mu *'}, \omega^{a*'}, \bar{\omega}^{a*'}$ exist.

Due to the fact that $\mathcal{S}_0$ has degree $+1$, there are no contributions to $t^4\Gamma^{(1)}$ from $\mathcal{S}_0$-invariants with dimension ≥ 5. Hence at one loop level $t^4\Gamma^{(1)}$ can be parameterized in terms of the following $\mathcal{S}_0$-invariants with dimension ≤ 4, whose coefficients are not fixed by the ST identities:

$$t^4\Gamma^{(1)} = \lambda_1^{(1)} \int d^4x\, G^a_{\mu\nu} G^{\mu\nu}_a + \rho_1^{(1)} \mathcal{S}_0 \left(\int d^4x\, \hat{A}^{a*'}_\mu A^a_\mu \right) + \rho_2^{(1)} \mathcal{S}_0 \left(\int d^4x\, \omega^{a*'} \omega^a \right). \quad (3.24)$$

$t^4\Gamma^{(1)}$ in eq. (3.24) fulfills the B-equation and the ghost equation. The anti-ghost equation in eq. (3.20) fixes the coefficient $\rho_2^{(1)}$ to zero. It does not constrain $\lambda_1^{(1)}, \rho_1^{(1)}$. These are free parameters entering into the most general solution for $t^4\Gamma^{(1)}$, compatible with the B-equation, the ghost equation, the anti-ghost equation and the one-loop order ST identities. The parameters $\lambda_1^{(1)}, \rho_1^{(1)}$ must be fixed by supplying a set of normalization conditions for $t^4\Gamma^{(1)}$. As an example, one might choose

$$\xi^{(1)}_{G^a_{\mu\nu} G^{\mu\nu}_a} = 0\,, \qquad \xi^{(1)}_{\hat{A}^{a*'}_\mu \partial^\mu \omega^a} = 0\,. \quad (3.25)$$

yielding

$$\lambda_1^{(1)} = 0\,, \qquad \rho_1^{(1)} = 0\,. \quad (3.26)$$

In order to study the dependence of higher-order non-symmetric counterterms on the normalization conditions, we do not restrict ourselves to a particular set of normalization conditions. We will thus keep the explicit dependence on $\rho_1^{(1)}, \lambda_1^{(1)}$. Finally we get

$$t^4\Gamma^{(1)} = \lambda_1^{(1)} \int d^4x\, G^a_{\mu\nu} G^{\mu\nu}_a + \rho_1^{(1)} \mathcal{S}_0 \left(\int d^4x\, \hat{A}^{a*'}_\mu A^a_\mu \right). \quad (3.27)$$

Only symmetric counterterms enter into $t^4\Gamma^{(1)}$. This is due to the fact that the only dimensionful parameter m enters into $\Gamma^{(0)}$ via a s-exact term. In theories where dimensionful parameters appear via a cohomologically non-trivial term (like for instance in the case of spontaneously broken models) this property is not true [35]. Also in Yang-Mills theory this feature is in general violated at higher orders, as we will discuss in the next subsection. It can be preserved only for a special choice of normalization conditions for $t^4\Gamma^{(1)}$.

3.2 Higher loops

At higher orders one also needs to take into account the contributions from lower order terms, entering into $\tilde{\zeta}_{\mathbf{0}}^{(n)}$ in eq. (2.29). We can limit ourselves to the terms of dimension ≤ 5 of the functional

$$\Delta^{(n)} = \sum_{j=1}^{n-1} \left(\frac{\delta\Gamma^{(n-j)}}{\delta \hat{A}^{a*'}_\mu} \frac{\delta\Gamma^{(j)}}{\delta A^a_\mu} + \frac{\delta\Gamma^{(n-j)}}{\delta \omega^{a*'}} \frac{\delta\Gamma^{(j)}}{\delta \omega^a} + \frac{\delta\Gamma^{(n-j)}}{\delta \bar{\omega}^{a*'}} \frac{\delta\Gamma^{(j)}}{\delta \bar{\omega}^a} \right). \quad (3.28)$$

$\Delta^{(n)}$ has ghost number $+1$. By taking into account the B-equation, the ghost equation the anti-ghost equation and the fact that $\mathrm{I\!\Gamma}^{(j)}$ is independent of $\bar{\rho}$, the part of dimension ≤ 5 of $\Delta^{(n)}$ (which from now on we will denote by $\Delta^{(n)}$ itself) reduces to

$$
\begin{aligned}
\Delta^{(n)} &= \sum_{j=1}^{n-1} \int d^4x \left(\rho_1^{(n-j)} \rho_1^{(j)} (-\partial^\mu \omega^a) \frac{\delta}{\delta A^a_\mu} \mathcal{S}_0 \left(\int d^4y\, \hat{A}^{b*'}_\nu A^{\nu b} \right) + \right. \\
&\qquad \left. + \rho_1^{(n-j)} \lambda_1^{(j)} (-\partial^\mu \omega^a) \frac{\delta}{\delta A^a_\mu} \left(\int d^4y\, G^{\rho\sigma b} G^b_{\rho\sigma} \right) \right) \\
&= - \int d^4x\, \partial^\mu \omega^a \frac{\delta}{\delta A^a_\mu} \left[\sum_{j=1}^{n-1} \rho_1^{(n-j)} \rho_1^{(j)} \mathcal{S}_0 \left(\int d^4y\, \hat{A}^{b*'}_\nu A^{\nu b} \right) + \right. \\
&\qquad \left. + \sum_{j=1}^{n-1} \rho_1^{(n-j)} \lambda_1^{(j)} \int d^4y\, G^{\rho\sigma b} G^b_{\rho\sigma} \right] .
\end{aligned}
\tag{3.29}
$$

The functional $\Delta^{(n)}$ fulfills the Wess-Zumino consistency condition

$$
\mathcal{S}_0(\Delta^{(n)}) = 0 . \tag{3.30}
$$

In order to determine the non-symmetric higher order counterterms we need to find a functional $\tilde{\zeta}^{(n)}_{\mathbf{0}}$ such that

$$
\mathcal{S}_0(\tilde{\zeta}^{(n)}_{\mathbf{0}}) = \Delta^{(n)} . \tag{3.31}
$$

We first consider the second term in the r.h.s. of eq. (3.29). We set

$$
\mathcal{H}_2 = \sum_{j=1}^{n-1} \rho_1^{(n-j)} \lambda_1^{(j)} \int d^4y\, G^{\rho\sigma b} G^b_{\rho\sigma} . \tag{3.32}
$$

$\mathcal{H}_2$ does not depend on the antifields. We first notice that the following formula holds for any functional H:

$$
\begin{aligned}
s \left(\int d^4x\, \partial_\mu \omega_a \frac{\delta H}{\delta A^a_\mu} \right) &= \int d^4x \left(-f^{abc} \partial_\mu \omega^b \omega^c \frac{\delta H}{\delta A^a_\mu} - \partial_\mu \omega^a f^{abc} \omega^c \frac{\delta H}{\delta A^b_\mu} \right) - \\
&\quad - \int d^4x\, \partial_\mu \omega^a \frac{\delta}{\delta A^a_\mu} (sH) \\
&= - \int d^4x\, \partial_\mu \omega^a \frac{\delta}{\delta A^a_\mu} (sH) .
\end{aligned}
\tag{3.33}
$$

Since $s\mathcal{H}_2 = 0$ we get that the second term in the r.h.s. of eq. (3.29) separately satisfies the Wess-Zumino consistency condition [21, 7]

$$
\mathcal{S}_0 \int d^4x \left(-\partial_\mu \omega_a \frac{\delta}{\delta A^a_\mu} (\mathcal{H}_2) \right) = s \int d^4x \left(-\partial_\mu \omega_a \frac{\delta}{\delta A^a_\mu} (\mathcal{H}_2) \right) = 0 . \tag{3.34}
$$

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Moreover one gets that

$$
\begin{aligned}
s\left(\int d^4x\, A^a_\mu \frac{\delta H}{\delta A^a_\mu}\right) &= \int d^4x \left(\partial_\mu \omega_a \frac{\delta H}{\delta A^a_\mu} + f^{abc} A^b_\mu \omega^c \frac{\delta H}{\delta A^a_\mu} + A^a_\mu s \frac{\delta H}{\delta A^a_\mu}\right) \\
&= \int d^4x \left(\partial_\mu \omega_a \frac{\delta H}{\delta A^a_\mu} + f^{abc} A^b_\mu \omega^c \frac{\delta H}{\delta A^a_\mu} + A^a_\mu \left[s, \frac{\delta}{\delta A^a_\mu}\right] H + A^a_\mu \frac{\delta}{\delta A^a_\mu}(sH)\right) \\
&= \int d^4x \left(\partial_\mu \omega_a \frac{\delta H}{\delta A^a_\mu} + A^a_\mu \frac{\delta}{\delta A^a_\mu}(sH)\right). \qquad (3.35)
\end{aligned}
$$

Since $s\mathcal{H}_2 = 0$, the above formula allows to write

$$
\int d^4x\, \partial_\mu \omega_a \frac{\delta \mathcal{H}_2}{\delta A^a_\mu} = s \int d^4x\, A^a_\mu \frac{\delta}{\delta A^a_\mu} \mathcal{H}_2 = \mathcal{S}_0 \int d^4x\, A^a_\mu \frac{\delta}{\delta A^a_\mu} \mathcal{H}_2 \,. \qquad (3.36)
$$

This gives the terms in $\tilde{\zeta}^{(n)}_{\mathbf{0}}$ generating the second term in the r.h.s. of eq. (3.29):

$$
\begin{aligned}
&-\int d^4x\, \partial^\mu \omega^a \frac{\delta}{\delta A^a_\mu} \left(\sum_{j=1}^{n-1} \rho_1^{(n-j)} \lambda_1^{(j)} \int d^4y\, G^{\rho\sigma b} G^b_{\rho\sigma}\right) = \\
&\qquad = -\mathcal{S}_0 \left(\int d^4x A^a_\mu \frac{\delta}{\delta A^a_\mu} \left[\sum_{j=1}^{n-1} \rho_1^{(n-j)} \lambda_1^{(j)} \int d^4y\, G^{\rho\sigma b} G^b_{\rho\sigma}\right]\right). \qquad (3.37)
\end{aligned}
$$

The analysis of the first term in the r.h.s. of eq. (3.29)

$$
\mathcal{H}_1 = \sum_{j=1}^{n-1} \rho_1^{(n-j)} \rho_1^{(j)} \int d^4x \left((-\partial^\mu \omega^a) \frac{\delta}{\delta A^a_\mu} \mathcal{S}_0 \left(\int d^4y\, \hat{A}^{b*'}_\nu A^{\nu b}\right)\right) \qquad (3.38)
$$

can be carried out by using techniques of homological perturbation theory and is discussed in appendix A. The result is

$$
\begin{aligned}
\mathcal{H}_1 = -\sum_{j=1}^{n-1} \rho_1^{(n-j)} \rho_1^{(j)} \Bigg[&\mathcal{S}_0 \left(\int d^4x\, A^a_\mu \frac{\delta}{\delta A^a_\mu} \mathcal{S}_0 \left(\int d^4y\, \hat{A}^{b*'}_\nu A^{\nu b}\right)\right) - \\
&- \mathcal{S}_0 \left(\int d^4x\, \frac{1}{g^2} (\Box A^d_\rho - \partial_\rho (\partial A)^d) A^{\rho d}\right) + \\
&+ \mathcal{S}_0 \left(\int d^4x\, \frac{2}{g^2} f^{dlm} A^l_\sigma (\partial_\rho A^m_\sigma - \partial_\sigma A^m_\rho) A^d_\rho\right) - \\
&- \mathcal{S}_0 \left(\int d^4x\, \frac{1}{4g^2} f^{vqk} f^{krd} A^q_\sigma A^r_\sigma A^v_\rho A^d_\rho\right) \Bigg] \qquad (3.39)
\end{aligned}
$$

Therefore the full functional $\tilde{\zeta}^{(n)}_{\mathbf{0}}$ is

$$
\begin{aligned}
\tilde{\zeta}^{(n)}_{\mathbf{0}} = &-\int d^4x A^a_\mu \frac{\delta}{\delta A^a_\mu} \left[\sum_{j=1}^{n-1} \rho_1^{(n-j)} \lambda_1^{(j)} \int d^4y\, G^{\rho\sigma b} G^b_{\rho\sigma}\right] - \\
&-\sum_{j=1}^{n-1} \rho_1^{(n-j)} \rho_1^{(j)} \int d^4x\, A^a_\mu \frac{\delta}{\delta A^a_\mu} \mathcal{S}_0 \left(\int d^4y\, \hat{A}^{b*'}_\nu A^{\nu b}\right) +
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_{j=1}^{n-1}\rho_1^{(n-j)}\rho_1^{(j)}\Big(\int d^4x\,\frac{1}{g^2}(\Box A^d_\rho-\partial_\rho(\partial A)^d)A^{\rho d}-\\
&\qquad\qquad -\int d^4x\,\frac{2}{g^2}f^{dlm}A^l_\sigma(\partial_\rho A^m_\sigma-\partial_\sigma A^m_\rho)A^d_\rho+\\
&\qquad\qquad +\int d^4x\,\frac{1}{4g^2}f^{vqk}f^{krd}A^q_\sigma A^r_\sigma A^v_\rho A^d_\rho\Big)\,.
\end{aligned}
\tag{3.40}
$$

The action-like part of the n-th order symmetric quantum effective action is then

$$
\begin{aligned}
t^4\mathrm{I\!\Gamma}^{(n)} &= \lambda_1^{(n)}\int d^4x\,G^a_{\mu\nu}G^{\mu\nu}_a+\rho_1^{(n)}\mathcal{S}_0\left(\int d^4x\,\hat{A}^{a*\prime}_\mu A^a_\mu\right)+\\
&+\int d^4x A^a_\mu\frac{\delta}{\delta A^a_\mu}\left[\sum_{j=1}^{n-1}\rho_1^{(n-j)}\lambda_1^{(j)}\int d^4y\,G^{\rho\sigma b}G^b_{\rho\sigma}\right]+\\
&+\sum_{j=1}^{n-1}\rho_1^{(n-j)}\rho_1^{(j)}\int d^4x\,A^a_\mu\frac{\delta}{\delta A^a_\mu}\mathcal{S}_0\left(\int d^4y\,\hat{A}^{b*\prime}_\nu A^{\nu b}\right)-\\
&-\sum_{j=1}^{n-1}\rho_1^{(n-j)}\rho_1^{(j)}\Big(\int d^4x\,\frac{1}{g^2}(\Box A^d_\rho-\partial_\rho(\partial A)^d)A^{\rho d}-\\
&\qquad\qquad -\int d^4x\,\frac{2}{g^2}f^{dlm}A^l_\sigma(\partial_\rho A^m_\sigma-\partial_\sigma A^m_\rho)A^d_\rho+\\
&\qquad\qquad +\int d^4x\,\frac{1}{4g^2}f^{vqk}f^{krd}A^q_\sigma A^r_\sigma A^v_\rho A^d_\rho\Big)\,.
\end{aligned}
\tag{3.41}
$$

Notice the appearance of non-symmetric counterterms in $t^4\mathrm{I\!\Gamma}^{(n)}$, due to the lower-order contributions in eq. (3.41). The non-symmetric counter-terms, depending on the lower order contributions, disappear if one chooses to impose the following normalization conditions for $t^4\mathrm{I\!\Gamma}^{(j)}$

$$
\lambda_1^{(j)}=0\,,\qquad \rho_1^{(j)}=0\,,\qquad j=1,2,\dots,n-1 \tag{3.42}
$$

equivalent to

$$
\xi^{(j)}_{G^a_{\mu\nu}G^{\mu\nu a}}=0\,,\qquad \xi^{(j)}_{\hat{A}^{a*\prime}_\mu\partial^\mu\omega_a}=0\,,\qquad j=1,2,\dots,n-1\,. \tag{3.43}
$$

They extend the one-loop normalization conditions in eq. (3.25).

4. Conclusions

In this paper we have analyzed the equivalence between the recursive restoration of the ST identities for anomaly-free gauge theories and the parameterization of the local approximation of the symmetric quantum effective action in terms of suitable ST functionals, associated with the cohomology classes of the classical linearized ST operator $\mathcal{S}_0$.

The ST functionals of dimension ≤ 4 correspond to the invariant counterterms, those of dimension > 4 generate the non-symmetric counterterms upon projection on the action-like sector. At orders higher than one in the loop expansion there are additional contributions to the non-invariant counterterms, arising from known lower order terms. They can also be parameterized by using the ST functionals.

We have applied the method to Yang-Mills theory in the Landau gauge with an explicit mass term introduced in a BRST invariant way via the BRST doublet $(\bar{\rho}, m)$ and we have derived the most general solution for the action-like part of the quantum effective action, compatible with the fulfillment of the ST identities, the ghost equation, the anti-ghost equation and the B-equation, to all orders in the loop expansion. By exploiting the ST parameterization we have also provided the full dependence of the solution on the normalization conditions, that have to be supplemented in order to fix the parameters left free in the symmetric quantum effective action after the imposition of the ST identities and of the other relevant symmetries of the model.

The method can applied whenever the classical linearized ST operator $\mathcal{S}_0$ is nilpotent. The cohomology classes of $\mathcal{S}_0$ in local jet spaces without power-counting restrictions have to be known in order to explicitly work out the form of the ST functionals and hence to solve the parameterization problem. Such a cohomology problem can be treated for a large class of theories of physical interest by applying the powerful methods of homological perturbation theory.

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A. ST parameterization of $\mathcal{H}_1$

In this appendix we analyze the term

$$\mathcal{H}_1 = \sum_{j=1}^{n-1} \rho_1^{(n-j)} \rho_1^{(j)} \int d^4x \, (-\partial^\mu \omega^a) \frac{\delta}{\delta A^a_\mu} \mathcal{S}_0 \left(\int d^4y \, \hat{A}^{b*'}_\nu A^{\nu b} \right) \tag{A.1}$$

contributing to the n-th order ST identities and construct a functional $\tilde{\zeta}^{(n)}_{\mathcal{H}_1}$ such that

$$\mathcal{H}_1 = \mathcal{S}_0(\tilde{\zeta}^{(n)}_{\mathcal{H}_1}) \, . \tag{A.2}$$

In order to construct $\tilde{\zeta}^{(n)}_{\mathcal{H}_1}$ we first notice the following relation:

$$\begin{aligned}\int d^4x \, \partial_\mu \omega^a \frac{\delta}{\delta A^a_\mu} \mathcal{S}_0 \left(\int d^4y \, \hat{A}^{b*'}_\nu A^{\nu b} \right) = {} & \mathcal{S}_0 \left(\int d^4x \, A^\mu_a \frac{\delta}{\delta A^a_\mu} \mathcal{S}_0 \left(\int d^4y \, \hat{A}^{b*'}_\nu A^{\nu b} \right) \right) + \\ & + \int d^4x d^4y \, A^a_\mu(x) \frac{\delta^2 \Gamma^{(0)}_{m=0}}{\delta A^a_\mu(x) \delta A^d_\rho(y)} \frac{\delta}{\delta \hat{A}(y)^{d*'}_\rho} \times \\ & \times \mathcal{S}_0 \left(\int d^4z \, \hat{A}^{b*'}_\nu A^{\nu b} \right) .\end{aligned} \tag{A.3}$$

The r.h.s. in the first line of the above equation is $\mathcal{S}_0$-exact. Therefore the analysis is reduced to the functional

$$
\begin{aligned}
\Delta' &= \int d^4x d^4y\, A^a_\mu(x) \frac{\delta^2 \Gamma^{(0)}_{m=0}}{\delta A^a_\mu(x) \delta A^d_\rho(y)} \frac{\delta}{\delta \hat{A}(y)^{d*'}_\rho} \mathcal{S}_0 \left(\int d^4z\, \hat{A}^{b*'}_\nu A^{\nu b} \right) \\
&= - \int d^4x d^4y\, A^a_\mu(x) \frac{\delta^2 \Gamma^{(0)}_{m=0}}{\delta A^a_\mu(x) \delta A^d_\rho(y)} \partial^\rho \omega^d \,. \qquad \text{(A.4)}
\end{aligned}
$$

We remark that Δ' is independent of the external sources and satisfies the Wess-Zumino consistency condition

$$
\mathcal{S}_0(\Delta') = s\Delta' = 0 \,. \qquad \text{(A.5)}
$$

Δ' depends only on A^a_μ and ω^a. In order to construct a functional $\tilde{\zeta}'$ such that

$$
s\tilde{\zeta}' = \Delta' \qquad \text{(A.6)}
$$

we use the techniques of homological perturbation theory [34]. We consider the grading induced by the counting operator for the fields A^a_μ

$$
\mathcal{N} = \int d^4x\, A^a_\mu \frac{\delta}{\delta A^a_\mu} \,. \qquad \text{(A.7)}
$$

The restriction of s to the subspace spanned by A^a_μ, ω^a and their derivatives decomposes according to this grading into

$$
s = s_{-1} + s_0 \,, \qquad \text{(A.8)}
$$

where

$$
\begin{aligned}
s_{-1} A^a_\mu &= \partial_\mu \omega_a \,, \qquad & s_{-1}\omega_a &= 0 \,, \\
s_0 A^a_\mu &= f^{abc} A^b_\mu \omega^c \,, \qquad & s_0 \omega^a &= -\frac{1}{2} f^{abc} \omega^b \omega^c \,. \qquad \text{(A.9)}
\end{aligned}
$$

The differentials s_{-1}, s_0 satisfy

$$
s_0^2 = s_{-1}^2 = 0 \,, \qquad \{s_{-1}, s_0\} = 0 \,, \qquad \text{(A.10)}
$$

as a consequence of the nilpotency of s. We notice that s_{-1} is the abelian approximation to s. s_{-1} admits the homotopy operator [37]

$$
\kappa = \int_0^1 dt \left(A^a_\mu \lambda_t \frac{\partial}{\partial(\partial_\mu \omega^a)} + \partial_{(\mu} A^a_{\nu)} \lambda_t \frac{\partial}{\partial(\partial_\mu \partial_\nu \omega^a)} + \partial_{(\mu} \partial_\nu A^a_{\sigma)} \lambda_t \frac{\partial}{\partial(\partial_\mu \partial_\nu \partial_\sigma \omega^a)} + \cdots \right) \qquad \text{(A.11)}
$$

where $(\mu, \ldots, \sigma)$ denotes total symmetrization and the operator λ_t acts as

$$
\begin{aligned}
&\lambda_t X \left(\omega^a, F^a_{\mu\nu}, \partial^\sigma F^a_{\mu\nu}, \partial^\sigma \omega^a, A^\mu_a, \partial_{\mu_1} \ldots \partial_{\mu_{n-2}} \partial_{(\mu_{n-1}} A^a_{\mu_n)} \right) = \\
&\qquad = X \left(\omega^a, F^a_{\mu\nu}, \partial^\sigma F^a_{\mu\nu}, t\partial^\sigma \omega^a, tA^\mu_a, t\partial_{\mu_1} \ldots \partial_{\mu_{n-2}} \partial_{(\mu_{n-1}} A^a_{\mu_n)} \right) \qquad \text{(A.12)}
\end{aligned}
$$

where $F^a_{\mu\nu}$ is

$$
F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu \,. \qquad \text{(A.13)}
$$

We also expand Δ' according to the degree induced by $\mathcal{N}$. We get

$$\Delta' = \Delta'_1 + \Delta'_2 + \Delta'_3 \tag{A.14}$$

where

$$\begin{aligned}
\Delta'_1 &= -\frac{1}{g^2}\left(\Box A^d_\rho - \partial_\rho(\partial A)^d\right)\partial^\rho\omega^d\,, \\
\Delta'_2 &= -\frac{1}{g^2}\left(4f^{dlm}A^l_\sigma\partial_\sigma A^m_\rho + 2f^{dlm}(\partial A)^l A^m_\rho - 2f^{dqk}A^q_\sigma\partial_\rho A^k_\sigma\right)\partial^\rho\omega^d\,, \\
\Delta'_3 &= \frac{3}{g^2}f^{dqk}f^{krv}A^q_\sigma A^r_\sigma A^v_\rho\partial^\rho\omega^d\,.
\end{aligned} \tag{A.15}$$

We now prove that it is possible to solve the condition

$$s\tilde{\zeta}' = \Delta' \tag{A.16}$$

for $\tilde{\zeta}'$. Eq. (A.16) reads at order j in the grading induced by $\mathcal{N}$:

$$s_{-1}\tilde{\zeta}'_{j+1} + s_0\tilde{\zeta}'_j = \Delta'_j\,, \qquad j = 0,1,2,3\,. \tag{A.17}$$

At order zero eq. (A.17) is verified by setting

$$\tilde{\zeta}'_1 = \tilde{\zeta}'_0 = 0\,. \tag{A.18}$$

We assume that eq. (A.17) is verified up to order $n-1$. At order n eq. (A.17) gives

$$s_{-1}\tilde{\zeta}'_{n+1} = \Delta'_n - s_0\tilde{\zeta}'_n\,. \tag{A.19}$$

Since s_{-1} is nilpotent, there is a consistency condition on the r.h.s. of eq. (A.19):

$$s_{-1}(\Delta'_n - s_0\tilde{\zeta}'_n) = 0\,. \tag{A.20}$$

The above equation is verified since

$$\begin{aligned}
s_{-1}(\Delta'_n - s_0\tilde{\zeta}'_n) &= s_{-1}\Delta'_n + s_0 s_{-1}\tilde{\zeta}'_n \\
&= s_{-1}\Delta'_n + s_0\left(\Delta'_{n-1} - s_0\tilde{\zeta}'_{n-1}\right) \\
&= s_{-1}\Delta'_n + s_0\Delta'_{n-1} \\
&= 0\,.
\end{aligned} \tag{A.21}$$

In the second line of the above equation we have used the recursive condition that eq. (A.17) is verified at order $n-1$. In the last line we have used the fact that Δ' fulfills eq. (A.5).

Then we can solve eq. (A.19) by using the homotopy operator κ:

$$\tilde{\zeta}'_{n+1} = \kappa(\Delta'_n - s_0\tilde{\zeta}'_n)\,. \tag{A.22}$$

By applying this procedure we get

$$\begin{aligned}
\tilde{\zeta}' = &-\int d^4x\,\frac{1}{g^2}\left(\Box A^d_\rho - \partial_\rho(\partial A)^d\right)A^{\rho d} + \\
&+\int d^4x\,\frac{2}{g^2}f^{dlm}A^l_\sigma\left(\partial_\rho A^m_\sigma - \partial_\sigma A^m_\rho\right)A^d_\rho - \\
&-\int d^4x\,\frac{1}{4g^2}f^{vqk}f^{krd}A^q_\sigma A^r_\sigma A^v_\rho A^d_\rho\,.
\end{aligned} \tag{A.23}$$

The final result for $\mathcal{H}_1$, combining eq. (A.3) and eq. (A.23), is given in eq. (3.39).

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